Physics 200 60 Name KEY

Exam 3 – Momentum, Rotation and Gravity

November 20, 2007

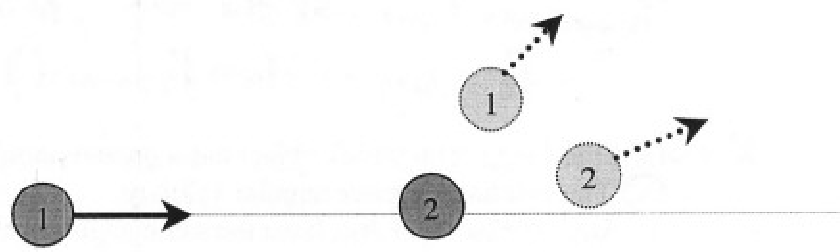
This is a closed book examination. You may use the provided concept/equation sheet. There is extra scratch paper available. Explanations must be included with all answers – even multiple-choice questions where the explanation is worth 75% of the possible points.

A general reminder about problem solving:

1. Draw a picture then create a simplified free body diagram with all forces
2. Write down what you know including coordinate frame
3. Write down what you don't know and/or want to know
4. List mathematical relationships
5. Simplify and solve
6. Check your answer – Is it reasonable? Are units correct?

- Show all work! Use extra paper if needed.

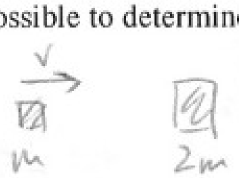
1. [4 PTS] A ball is moving in the $+\hat{i}$ direction when it collides with another stationary ball. After the collision both balls move off in different directions but both have $+\hat{j}$ components to their velocities. An example diagram is drawn below where the dashed objects represent objects after the collision. Is such a collision possible?
   A. No.
   B. Yes.



$\vec{P}_f = \vec{P}_i$

One ball would need to have a $-\hat{j}$ component.
(unless there is an outside force)

2. [8 PTS] A block collides with another stationary block that has twice the mass of the moving block. The blocks stick together after the collision. Assume the surface is frictionless. The final kinetic energy is
   A. 1/9 less than the initial kinetic energy.
   B. 1/3 less than the initial kinetic energy.
   C. the same as the initial kinetic energy.
   D. 3 times more than the initial kinetic energy.
   E. 9 times more than the initial kinetic energy.
   F. not possible to determine. More information is needed.



$P_i = P_f$ so $mv_i = (m+2m)v_f$

$\frac{1}{3}v_i = v_f$

$KE_i = \frac{1}{2}mv_i^2 + \frac{1}{2}(2m)0^2 = \frac{1}{2}mv_i^2$

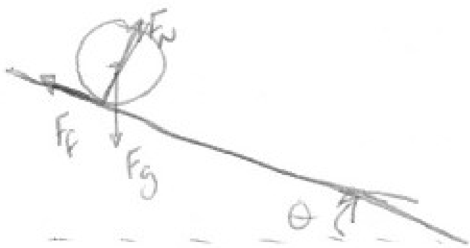
$KE_f = \frac{1}{2}(m+2m)v_f^2 = \frac{1}{2}3m\frac{1}{9}v_i^2 = \frac{1}{3}\frac{1}{2}mv_i^2$

$KE_f = \frac{1}{3}KE_i$

12

7. [8 PTS] Two solid disks are set to roll (without slipping) down an incline. The disks are identical except one disk has twice the radius of the other disk. When these two disks are released which one reaches the bottom of the incline first?
   A. The larger disk reaches the bottom first.
   B. Both disks reach the bottom at the same time.
   C. The smaller disk reaches the bottom first.

$I = \beta m r^2 \quad \omega = \frac{v}{r}$



Use energy conservation

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}(\beta m r^2)\left(\frac{v^2}{r^2}\right)$

$gh = \frac{1}{2}v^2 + \frac{1}{2}\beta v^2$

$\left(\frac{2gh}{1+\beta}\right)^{1/2} = v$ ∴ V is independent of both mass and radius

∴ shape of object does matter

$F_N - F_g\cos\theta = 0 \qquad a = r\alpha$

$\Sigma F = ma \qquad F_g\sin\theta - F_f = ma$

$\Sigma\tau = I\alpha \qquad rF_f = I\alpha = \beta m r^2\left(\frac{a}{r}\right) \qquad F_f = \beta m a$

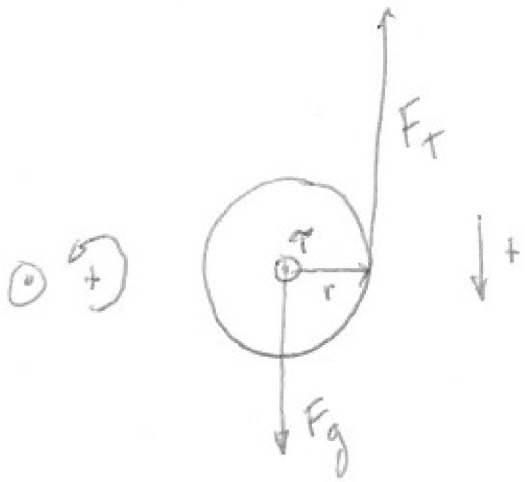
∴ $mg\sin\theta - \beta ma = ma \qquad a = \frac{g\sin\theta}{(1+\beta)}$ reduces to what you expect when $\beta = 0$ ∴ a is independent of mass and radius

8. [12 PTS] You have a spool of thread. You let the spool fall to the floor while holding onto the end of the thread. Assume the thread is massless, the spool is a solid cylinder (which has the same moment of inertia as a disk) and the thread is wound tightly around the spool.
   A. How long does it take the cylinder to hit the ground if you drop it from 2 meters?
   B. If you drop a ball at the same time, which would hit the ground first?

9. [12 PTS] A playground merry-go-round that is initially at rest is spun by a child. The child pushes with a force of 100N for 12 seconds such that the merry-go-round rotates in the ccw direction as viewed from above (the $+\hat{k}$ direction). The merry-go-round can be modeled as a solid disk with a radius of 2 meters and a mass of 500 kg. Assume the child pushes tangentially.
   A. Assume there is no friction slowing the rotation.
      i. How fast is the merry-go-round spinning after 12 seconds?
      ii. How fast is the child running after 12 seconds?
      iii. How far does the child run in 12 seconds?
   B. Assume there is a frictional torque of $-80\hat{k}$ Nm slowing the rotation.
      i. How fast is the merry-go-round spinning after 12 seconds?
      ii. How fast is the child running after 12 seconds?
      iii. How far does the child run in 12 seconds?

2/

$$\sum F = ma$$



$$F_g - F_T = ma$$

$$mg - F_T = ma$$

Spool = cylinder

$$\sum \tau = I\alpha \qquad r\alpha = a \qquad I = \frac{1}{2}mr^2$$

$$rF_T = I\alpha = \frac{1}{2}mr^2\left(\frac{a}{r}\right)$$

$$F_T = \frac{1}{2}ma$$

Spool is <u>not</u> in free-fall

$$mg - \frac{1}{2}ma = ma$$

$$\frac{g}{\left(1+\frac{1}{2}\right)} = a \qquad a = \frac{2}{3}g$$

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

$$h = y - y_0 = \frac{1}{2}\left(\frac{2}{3}g\right)t^2$$

$$\left(\frac{3h}{g}\right)^{1/2} = t \qquad \text{so} \quad t = \left(\frac{3 \cdot 2\,m}{9.81\,m/s^2}\right)^{1/2} = 0.78\ \text{sec}$$

A ball in free fall $a = g$

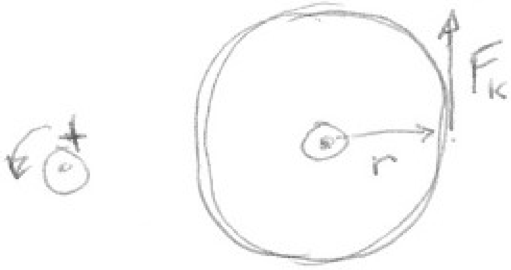
$$h = y - y_0 = \frac{1}{2}gt^2$$

$$t = \left(\frac{2h}{g}\right)^{1/2} = \left(\frac{2 \cdot 2\,m}{9.81\,m/s^2}\right)^{1/2} = 0.64\ \text{sec}$$

Ball hits first ... 0.14 seconds before spool a/

From top

No Friction



$$\sum_i \tau = I\alpha$$

$$rF_k = \frac{1}{2}mr^2\alpha$$

$$\frac{(rF_k)}{(\frac{1}{2}mr^2)} = \alpha = \frac{200\,Nm}{1000\,kgm^2}$$

$$\alpha = 0.2\ rad/s^2$$

$$\vec{\tau}_k = \vec{r}\times\vec{F} = (2m)(100N) = 200\,Nm$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2}500kg\,(2m)^2 = 1000\,kg\,m^2$$

$$\omega(t) = \int \alpha\,dt$$

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \int \omega(t)\,dt = \int \omega_0 + \alpha t\ dt$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$\theta_0 = 0\ rad$

$\omega_0 = 0\ rad/sec$

$t = 12sec$

(i) $\omega = \alpha t = 0.2\frac{rad}{s^2}\ 12sec = 2.4\frac{rad}{sec}$

(ii) $v = r\omega = 2m \cdot 2.4\frac{rad}{sec} = 4.8\ m/s$

(this is a tad fast)

(iii) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}0.2rad(12s)^2 = 14.4\ rad$

$s = r\theta = (2m)(14.4rad)$

$= 28.8m$

(Note: 14.4 rad = 2.3 rev)

Add Friction

$$\sum \tau = I\alpha$$

$$rF_k - 80Nm = I\alpha$$

$$\alpha = \frac{200Nm - 80Nm}{1000\ kgm^2}$$

$$\alpha = 0.12\ rad/s^2$$

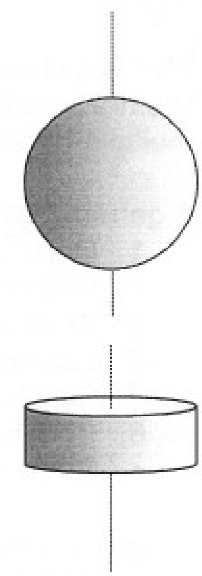
(i) $\omega = \alpha t = 0.12\frac{rad}{s} \cdot 12s = 1.44\frac{rad}{sec}$

(ii) $v = r\omega = 2.88\ m/s$

(iii) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}0.12\frac{rad}{s}(12s)^2 = 8.64\ rad$ $\quad s = r\theta = 17.3\ m$

(or 1.38 rev)

You decide to spin a solid ball and a solid disk (see picture below). Both the disk and ball have the same radius and mass and you spin both with the same torque for 10 seconds. The next three questions involve this ball and disk. NOTE: REMEMBER TO PROVIDE EXPLANATIONS.



3. [4 PTS] After being spun which object has more energy?
   A. The ball has more energy.
   B. Both the ball and disk have the same energy.
   C. The disk has more energy.

$KE = \frac{1}{2}I\omega^2 = \frac{L}{2I}$ $I_{ball} < I_{disk}$

$\therefore KE_{ball} > KE_{disk}$

4. [4 PTS] After being spun which object has a greater angular momentum?
   A. The ball has a greater angular momentum.
   B. Both the ball and disk have the same angular momentum.
   C. The disk has a greater angular momentum.

$\sum_i \vec{\tau} = \frac{d\vec{L}}{dt}$ so $\int \vec{\tau}_{net} \cdot dt = \Delta \vec{L}$

Torque and time are the same so $\Delta\vec{L}$ is the same
(both objects start from rest)

5. [4 PTS] After being spun which object has a greater angular velocity?
   A. The ball has a greater angular velocity.
   B. Both the ball and disk have the same angular velocity.
   C. The disk has a greater angular velocity.

$\vec{L} = I\vec{\omega}$ $I_{ball} = \frac{2}{5}mr^2$ $I_{Disk} = \frac{1}{2}mr^2$

$I_{ball}\,\omega_{ball} = I_{disk}\,\omega_{disk}$ $I_{ball} < I_{disk}$ so $\omega_{ball} > \omega_{disk}$

$\omega_{ball} = \frac{I_{disk}}{I_{ball}}\,\omega_{disk}$

6. [4 PTS] You find yourself gravitationally attracted to a large unidentified fluffy object (LUFO) that is located 4 meters in front of you. You wish to decrease this attractive force by a factor of 4.
   A. You should move 2 meters towards the LUFO.
   B. You should move 1 meter towards the LUFO.
   C. You should move 2 meters further away from the LUFO.
   D. You should move 4 meters further away from the LUFO.
   E. You should move 16 meters further away from the LUFO.

$F_i = G\frac{m_1 m_2}{r_i^2}$ $F_f = \frac{1}{4}F_i = \frac{G m_1 m_2}{4r_i^2}$ $r_f = 2r_i$

$r_f = 8m = 2(4m)$

Moving away decreases the force
factor of 2 in distance is factor of 4 in force ($\frac{1}{r^2}$ law)

16